Chapter 11
Surface Area & Volume

Lesson objectives: to discover the properties of surface area and volume and how to apply those properties to solve problems.

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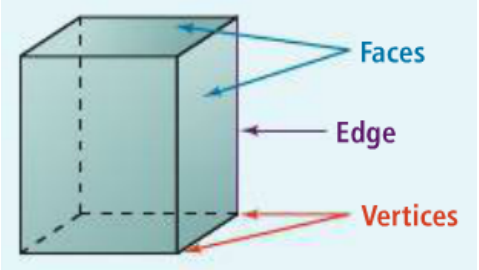
Vocabulary

A **polyhedron** is a space figure, or three-dimensional figure, whose surfaces are polygons. Each polygon is a **face** of the polyhedron. An **edge** is a segment that is formed by the intersection of two faces. A **vertex** is a point where three or more edges intersect.



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Surface Area vs Volume

Think: *Outside the box*

Think: *Inside the box*

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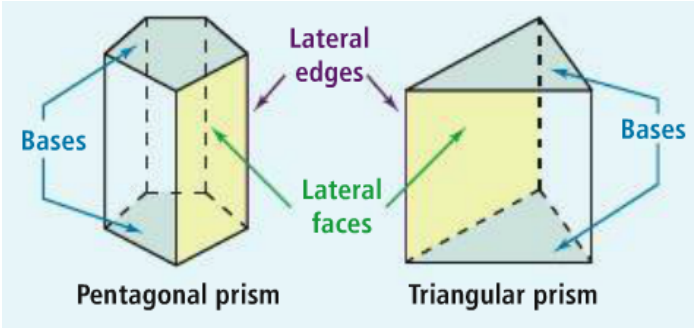
Surface areas of prisms and cylinders



A **prism** is a polyhedron with two congruent, parallel faces, called **bases.** The other faces are **lateral faces.** You can name a prism using the shape of its bases.

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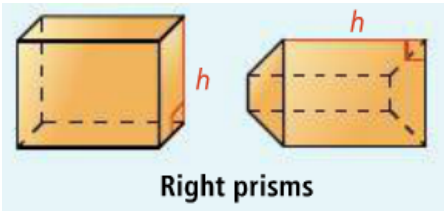
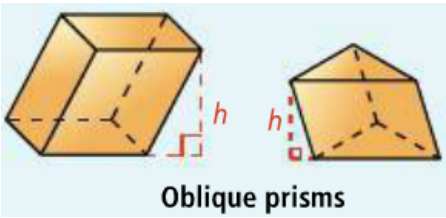


Right prisms



Oblique prisms

An **altitude** of a prism is a perpendicular segment that joins the planes of the bases. The **height** h of a prism is the length of an altitude. A prism may either be right or oblique.

In a **right prism,** the lateral faces are rectangles and a lateral edge is an altitude. In an **oblique prism,** some or all of the lateral faces are nonrectangular.

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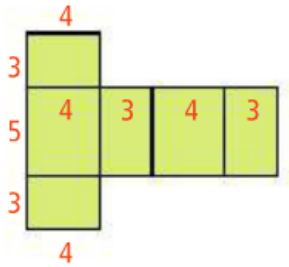
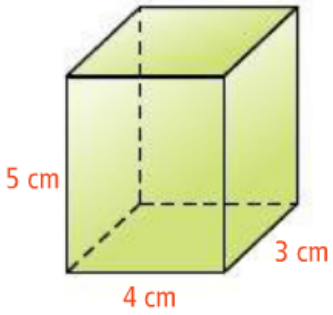
The **lateral area** (L.A.) of a prism is the sum of the areas of the lateral faces. The **surface area** (S.A.) is the sum of the lateral area and the area of the two bases.

What is the surface area of the prism at the right?





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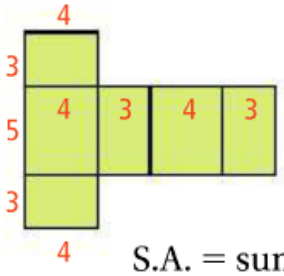
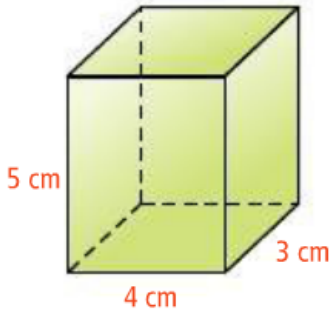
What is the surface area of the prism at the right?





S.A. = sum of areas of all the faces

$= 5 \cdot 4 + 5 \cdot 3 + 5 \cdot 4 + 5 \cdot 3 + 3 \cdot 4 + 3 \cdot 4$

$= 20 + 15 + 20 + 15 + 12 + 12$

$= 94$

The surface area of the prism is 94 cm^2.

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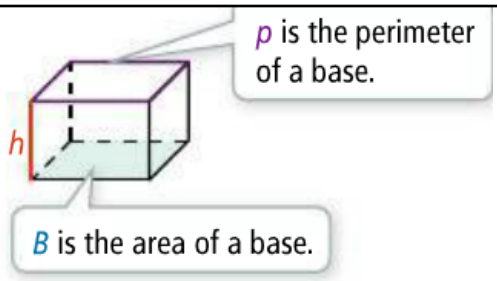
Theorem 11-1: lateral and surface area of a prism



The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

L.A. = ph

The surface area of a right prism is the sum of the lateral area and the areas of the two bases.

S.A. = L.A. + $2B$

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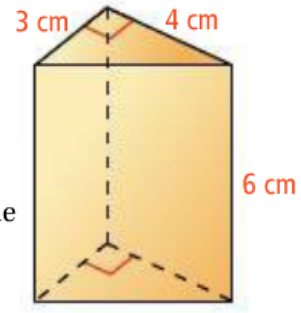
Example

What is the surface area of the prism at the right?



Step 1 Find the perimeter of a base.

The perimeter of the base is the sum of the side lengths of the triangle. Since the base is a right triangle, the hypotenuse is $\sqrt{3^2 + 4^2}$ cm, or 5 cm, by the Pythagorean Theorem.

$p = 3 + 4 + 5 = 12$

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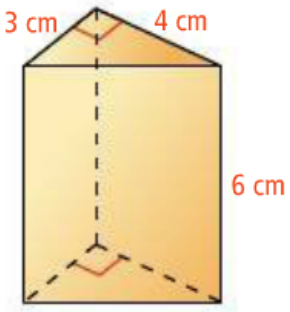
Example

What is the surface area of the prism at the right?



Step 2 Find the lateral area of the prism.

L.A. $= ph$ Use the formula for lateral area.

$= 12 \cdot 6$ Substitute 12 for p and 6 for h.

$= 72$ Simplify.

Step 3 Find the area of a base.

$B = \frac{1}{2}bh$ Use the formula for the area of a triangle.

$= \frac{1}{2}(3 \cdot 4)$ Substitute 3 for b and 4 for h.

$= 6$

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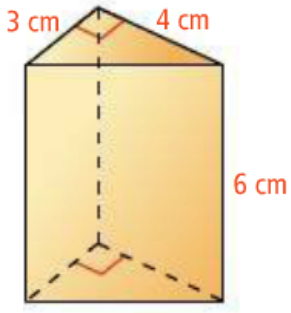
Example

What is the surface area of the prism at the right?



Step 4 Find the surface area of the prism.

S.A. $= \text{L.A.} + 2B$ Use the formula for surface area.

$= 72 + 2(6)$ Substitute 72 for L.A. and 6 for B.

$= 84$ Simplify.

The surface area of the prism is 84 cm^2.

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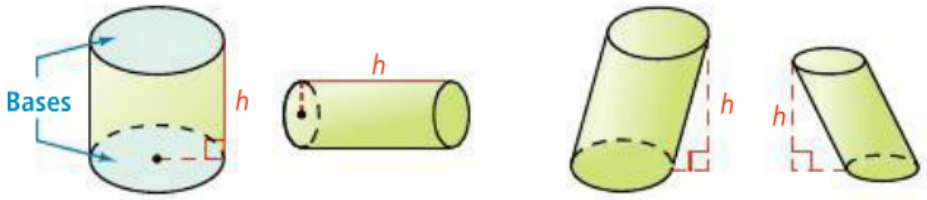

Right cylinders **Oblique cylinders**

A **cylinder** is a solid that has two congruent parallel **bases** that are circles. An **altitude** of a cylinder is a perpendicular segment that joins the planes of the bases. The **height** h of a cylinder is the length of an altitude.

In a **right cylinder,** the segment joining the centers of the bases is an altitude. In an **oblique cylinder,** the segment joining the centers is not perpendicular to the planes containing the bases.

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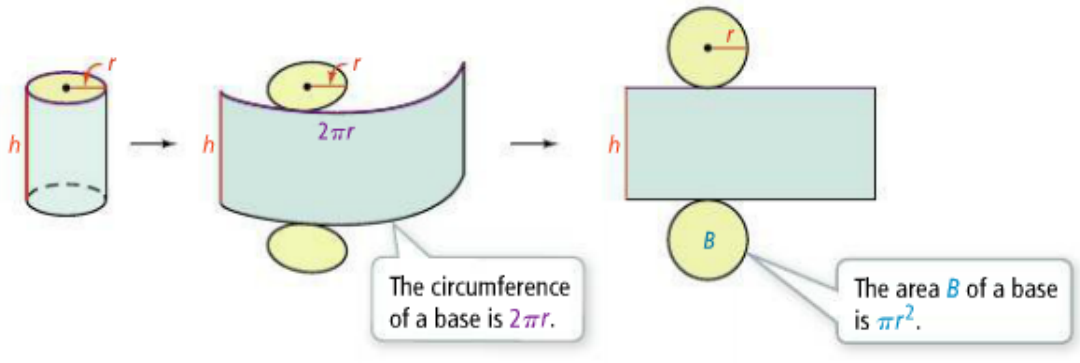
To find the area of the curved surface of a cylinder, visualize "unrolling" it. The area of the resulting rectangle is the **lateral area** of the cylinder. The **surface area** of a cylinder is the sum of the lateral area and the areas of the two circular bases. You can find formulas for these areas by looking at a net for a cylinder.



Lateral Area = $2\pi rh$ Surface Area = L.A. + $2\pi r^2$

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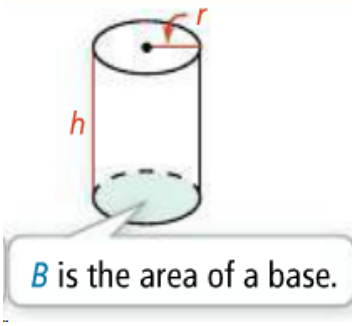
Theorem 11-2: lateral and surface areas of a cylinder



The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.

L.A. = $2\pi r \cdot h$, or L.A. = πdh

The surface area of a right cylinder is the sum of the lateral area and the areas of the two bases.

S.A. = L.A. + $2B$, or S.A. = $2\pi rh + 2\pi r^2$

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Multiple Choice The radius of the base of a cylinder is 4 in. and its height is 6 in. What is the surface area of the cylinder in terms of π?

(A) 32π in.^{2} (B) 42π in.^{2} (C) 80π in.^{2} (D) 120π in.^{2}

S.A. $= \text{L.A.} + 2B$	Use the formula for surface area of a cylinder.
$= 2\pi rh + 2(\pi r^2)$	Substitute the formulas for lateral area and area of a circle.
$= 2\pi(4)(6) + 2(\pi 4^2)$	Substitute 4 for r and 6 for h.
$= 48\pi + 32\pi$	Simplify.
$= 80\pi$	

The surface area of the cylinder is 80π in.^{2}. The correct choice is C.

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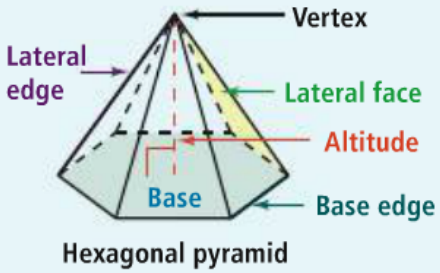
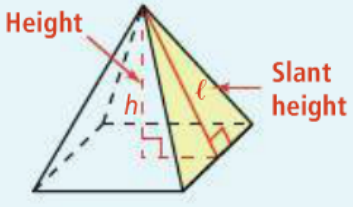
A **pyramid** is a polyhedron in which one face (the **base**) can be any polygon and the other faces (the **lateral faces**) are triangles that meet at a common vertex (called the **vertex** of the pyramid).

You name a pyramid by the shape of its base. The **altitude** of a pyramid is the perpendicular segment from the vertex to the plane of the base. The length of the altitude is the **height** h of the pyramid.

A **regular pyramid** is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles. The **slant height** ℓ is the length of the altitude of a lateral face of the pyramid.



Hexagonal pyramid



Square pyramid

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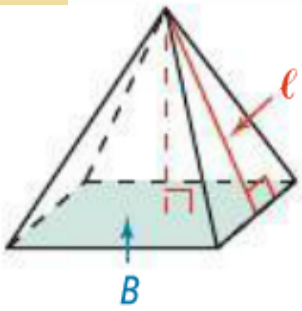
Theorem 11-3 Lateral and Surface Areas of a Pyramid

The lateral area of a regular pyramid is half the product of the perimeter p of the base and the slant height ℓ of the pyramid.

$$\text{L.A.} = \frac{1}{2}p\ell$$

The surface area of a regular pyramid is the sum of the lateral area and the area B of the base.

$$\text{S.A.} = \text{L.A.} + B$$



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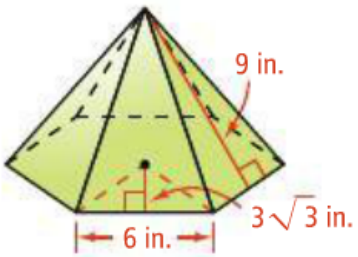
What is the surface area of the hexagonal pyramid?



$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area.
$= \frac{1}{2}p\ell + \frac{1}{2}ap$	Substitute the formulas for L.A. and B.
$= \frac{1}{2}(36)(9) + \frac{1}{2}\left(3\sqrt{3}\right)(36)$	Substitute.
≈ 255.5307436	Use a calculator.

The surface area of the pyramid is about 256 in.^{2}.

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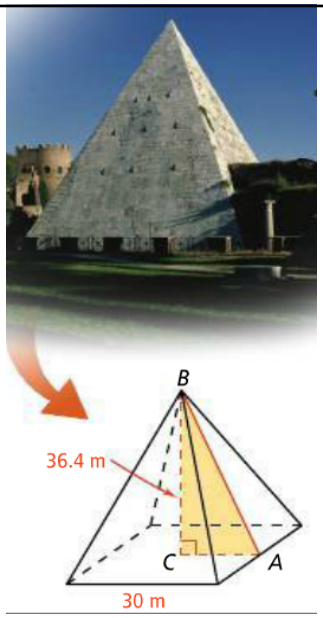
Social Studies The Pyramid of Cestius is located in Rome, Italy. Using the dimensions in the figure below, what is the lateral area of the Pyramid of Cestius? Round to the nearest square meter.



Step 1 Find the perimeter of the base.

$p = 4s$ Use the formula for the perimeter of a square.

$= 4 \cdot 30$ Substitute 30 for s.

$= 120$ Simplify.

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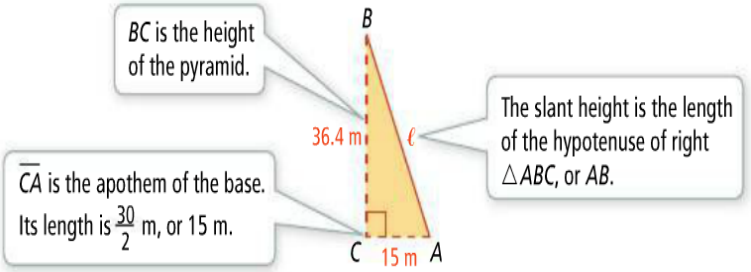
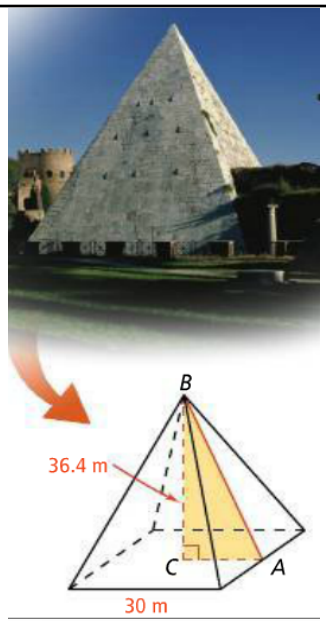
Step 2 Find the slant height of the pyramid.





$\ell = \sqrt{CA^2 + BC^2}$ Use the Pythagorean Theorem.

$= \sqrt{15^2 + 36.4^2}$ Substitute 15 for CA and 36.4 for BC.

$= \sqrt{1549.96}$ Simplify.

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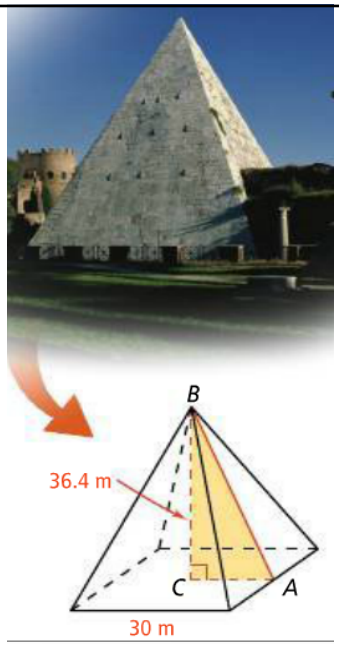
Step 3 Find the lateral area of the pyramid.

L.A. $= \frac{1}{2}p\ell$ Use the formula for lateral area.

$= \frac{1}{2}(120)\sqrt{1549.96}$ Substitute 120 for p and $\sqrt{1549.96}$ for ℓ.

≈ 2362.171882 Use a calculator.

The lateral area of the Pyramid of Cestius is about 2362 m^2.



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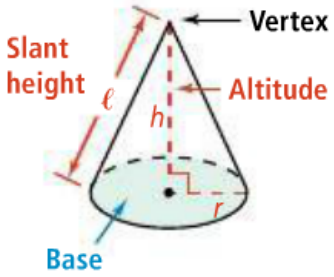
Like a pyramid, a **cone** is a solid that has one base and a vertex that is not in the same plane as the base. However, the **base** of a cone is a circle. In a **right cone**, the **altitude** is a perpendicular segment from the **vertex** to the center of the base. The **height** h is the length of the altitude. The **slant height** ℓ is the distance from the vertex to a point on the edge of the base. In this book, you can assume that a cone is a right cone unless stated or pictured otherwise.



The **lateral area** is half the circumference of the base times the slant height. The formulas for the lateral area and **surface area** of a cone are similar to those for a pyramid.

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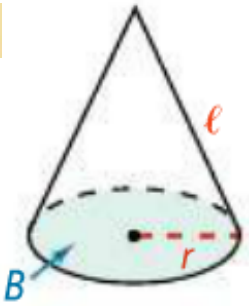
Theorem 11-4 Lateral and Surface Areas of a Cone



The lateral area of a right cone is half the product of the circumference of the base and the slant height of the cone.

$$\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot \ell, \text{ or L.A.} = \pi r \ell$$

The surface area of a cone is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$

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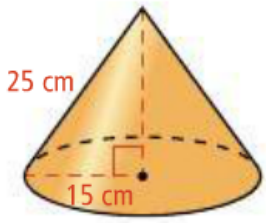
What is the surface area of the cone in terms of π?



$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area.
$= \pi r\ell + \pi r^2$	Substitute the formulas for L.A. and B.
$= \pi(15)(25) + \pi(15)^2$	Substitute 15 for r and 25 for ℓ.
$= 375\pi + 225\pi$	Simplify.
$= 600\pi$	

The surface area of the cone is 600π cm^2.

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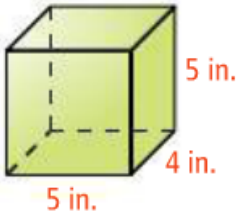
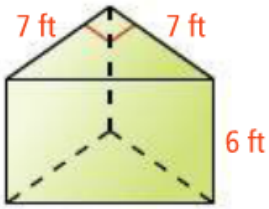
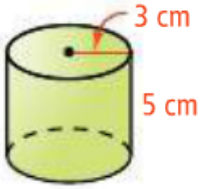
What is the surface area of each prism?

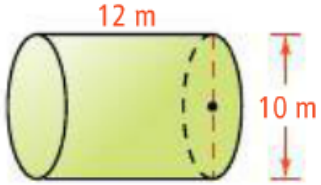
1. 

2. 

Exit Slip!

What is the surface area of each cylinder?

3. 

4. 

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